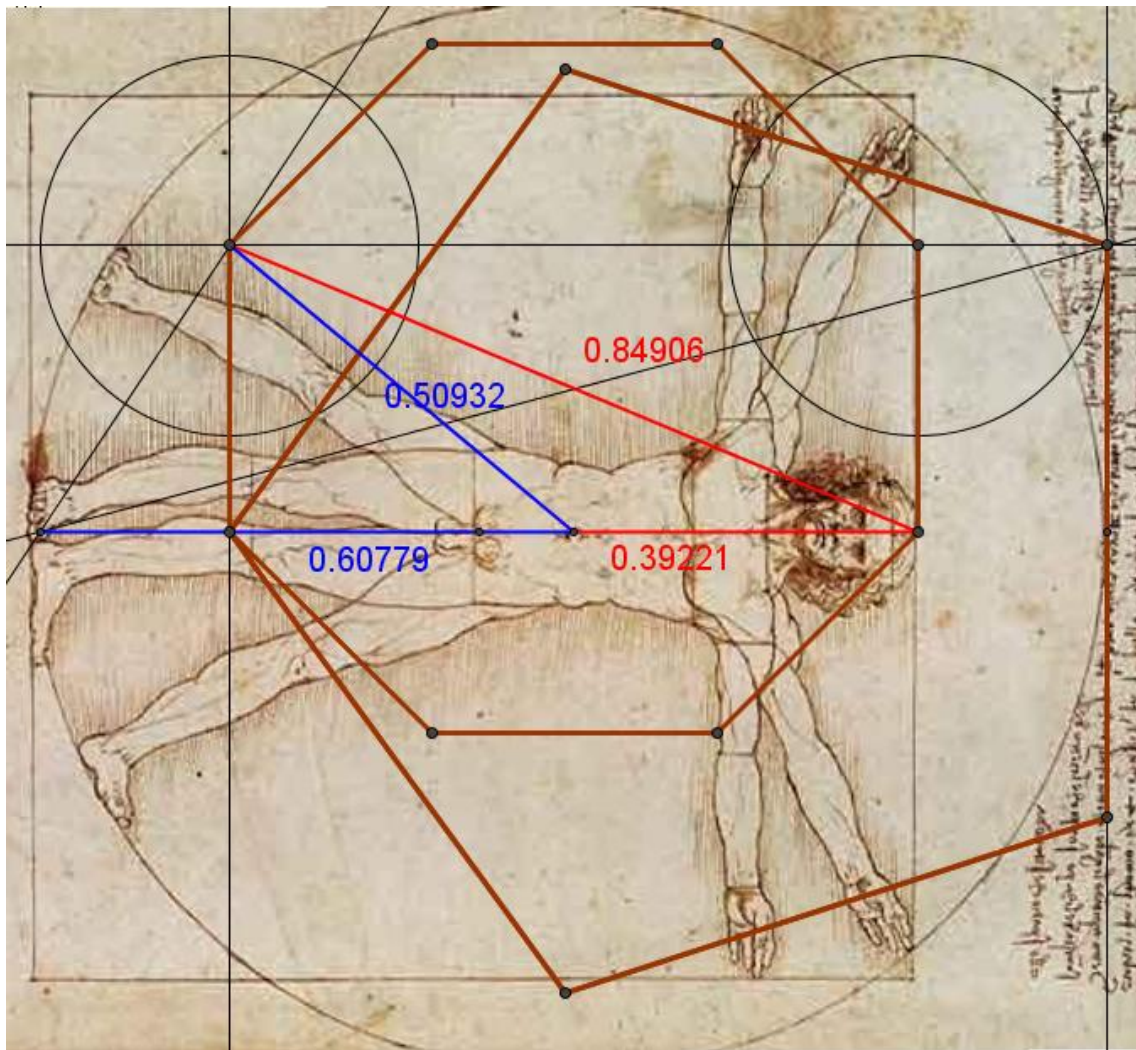


The canon of da Vinci can be built of both of a pentagon (the side = 2X) and an octagon (the side= X).

The height of the square of the canon of da Vinci is $X / \tan 18^\circ$, and the height of circle of the canon of da Vinci is $2X / \tan 18^\circ$ minus $X / \tan 22.5^\circ$. The height of square written above is derived as described below. The ratio between the length of a side of the regular pentagon and the height of the same pentagon is 3.077, because $\tan 18^\circ$ is $1/3.077$ where 18 is the angle (in degree) between the horizontal line and right-upper side of pentagon as displayed in the following figure. Then, if half side of the pentagon is X, the height of the pentagon is 3.077 X. The ratio between the height of the da Vinci square and the radius of the da Vinci circle must be closer to 0.6035 (derived of the square rotation) than 0.618 (golden proportion) as seen in the figure. Here, we intend to treat the radius as 0.6077 in the middle value. Then the diameter of the circle is the double of 0,6077 (1.2155) times the height of the square.

The ratio between the length of a side of the regular octagon and the height of the same octagon is 2.414, because $\tan 22.5^\circ$ is $1/2.414$ where 22.5 is the angle formed with the vertical and the straight line that cuts the one of upper vertexes and one of the lower vertexes of the octagon. Then, if a side of the octagon is X too (like half side of the pentagon), the ratio between the height of the pentagon and the height of the octagon is 0.7844. As the height of the pentagon is equal to the height of the da Vinci square and equals to 1, the first equation is $3.077 X = 1$. Then, the diameter of the circle is 1.2155 and the height of the octagon is 0.7844, (and two times the height of pentagon minus one time the height of the octagon is $2 - 0.7844 = 1.2155$ which equals the diameter of the circle of da Vinci)

Two times the height of pentagon minus one time the height of the octagon is $2 * 3.077X - 2.414X$ (where X always is a side of the octagon and half side of the pentagon), then, the height of circle of the canon of da Vinci is $2X / \tan 18^\circ - X / \tan 22.5^\circ = 2 * 3.077X - 2.414X = 1.2155$ where $\tan 18^\circ = 1/3.077$ and $\tan 22.5^\circ = 2.414$.



If the human height is 1 the distance feet-navel is 0.60779 and the distance navel-head is 0.39221. The ratio of the two distances is close to square root of $1/5$ and square root of $1/12$. If the human height is 0.73580617104571988103260342231771 we have exactly square root of $1/5$ for the distance feet-navel, if the human height is 0.73601666661253225601030826354516 we have exactly square root of $1/12$ for the distance navel-head.

If the human height is 1 the distance octagon vertex-navel is 0.50931686100588 and the distance octagon vertex-head is 0.849056083340138. The ratio of the two distances is close to $3/5$. If the human height is 0.73628035651399860391367975585043 we have exactly $3/8$ for the distance octagon vertex-navel, if the human height is 0.73611156231433593700255770155035 we have exactly $5/8$ for the distance octagon vertex-head.

Now, I found 0.736 in the average value of the human height for a study of an Indonesian team about Hugelschaffer's Egg-Shaped Curve measuring 63 hen's eggs.

The Constructions of Egg-Shaped Surface Equations using Hugelschaffer's Egg-Shaped Curve

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here was where i found the hugelschaffer egg equation they measured 63 eggs too

The corresponding between parameter a and b and parameter a and w to the oval egg-shaped surface are $0,681 \leq \frac{b}{a} \leq 0,815$ and $0,0292 \leq \frac{w}{a} \leq 0,1675$. This interval is measured from 63 chicken eggs.

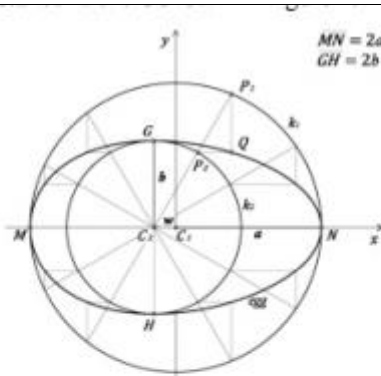


Fig. 1. Construction of Hugelschaffer's egg-shaped curve.

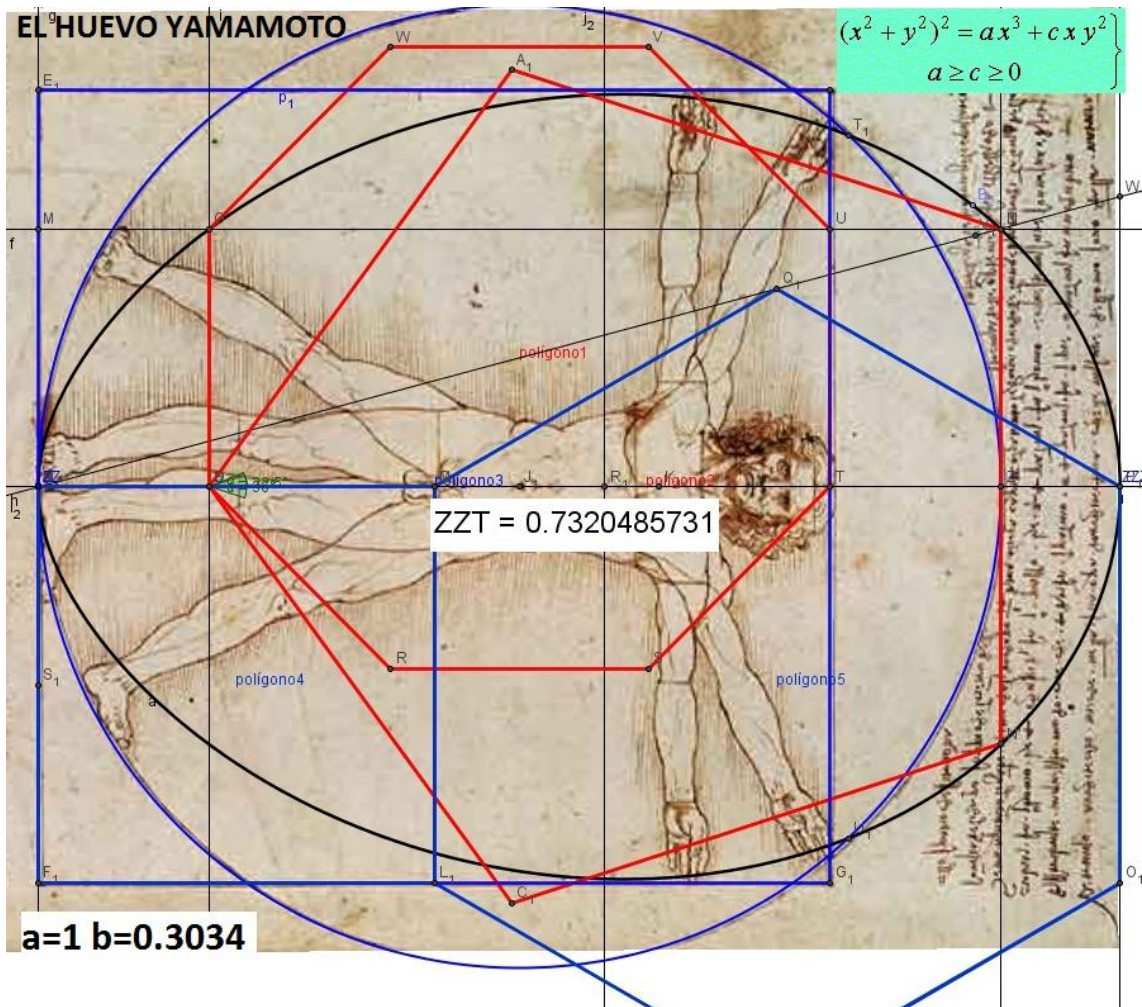
Furthermore, by using the comparison of gradient C_2P_2 and C_2P_1 , we obtained the equation of Hugelchaffer's egg-shaped curve, that is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} \left(1 + \frac{2wx + w^2}{a^2} \right) = 1, \quad (1)$$

with $a > b > 0$ and $0 < |w| < a$; a, b, w constants and MN is major axis of curve and GH is minor axis of curve [7].

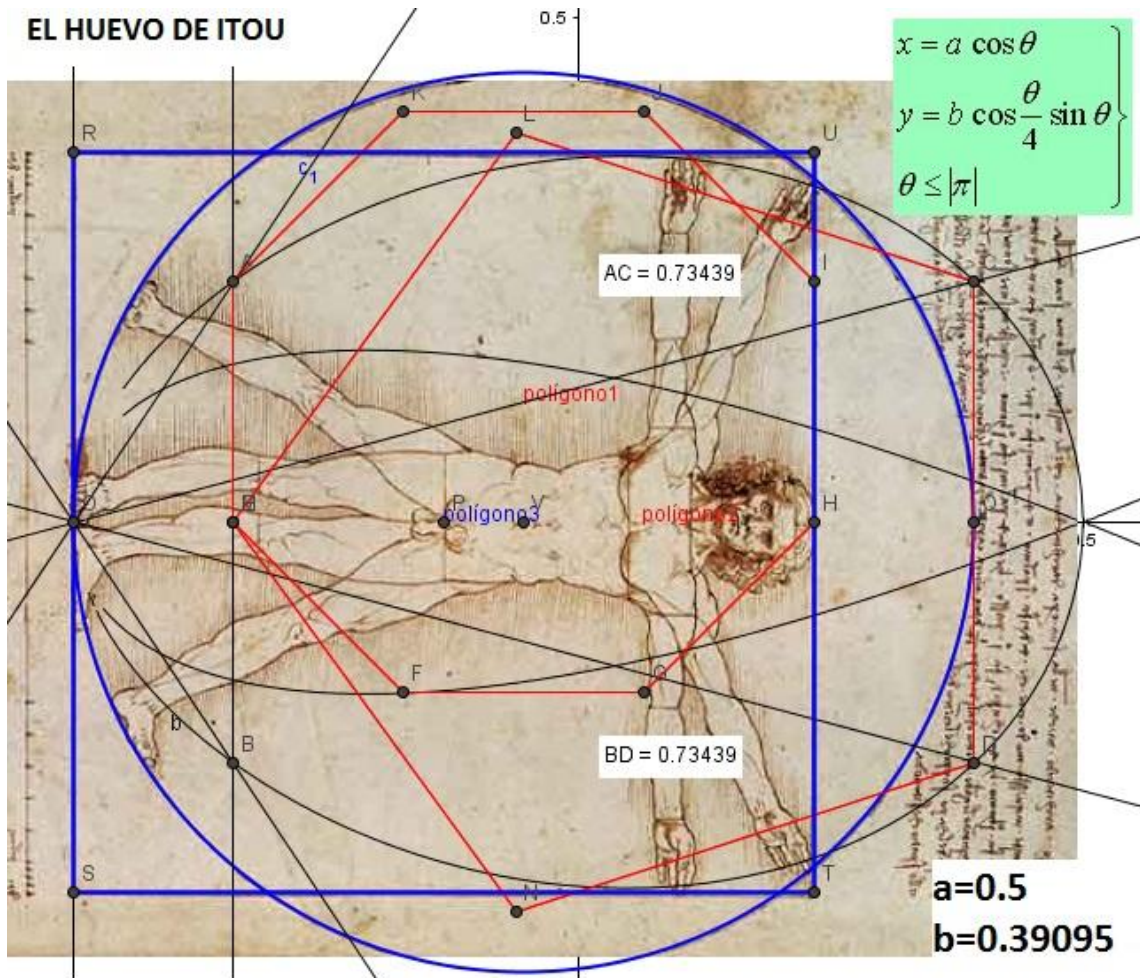
For $a=0.5$, b is between 0.3405 and 0.4075, while w is between 0.0146 and 0.08375, but the necessary condition in my egg on the anatomical canon is that the egg curve pass on a pentagon vertex and a octagon vertex (the vertex of the distances $3/8$ and $5/8$), then I have b is between 0.3405 and 0.3705 and w is between 0.03645 and 0.08375. This discards a half of the measured eggs. With these values the human height (when the egg height is 1) is between 0.722984 and 0.749273. the average height is 0.736128

The equations of egg shape curves on the Nobuo Yamamoto web with the necessary condition gives us similar values, values where the distances are close to $3/8$ $5/8$ square root $1/5$ and square root $1/12$



the egg major axis is 1 and the human height 0.7320478 that is close platine number= square root of 3 -1 (blue square and blue hexagon) Nabuo Yamanoto gives us $a=1$ $c=0.3$ for a hen's egg but is $a=1$ $c=0.3034$ for the necessary condition (egg passing on vertexes)

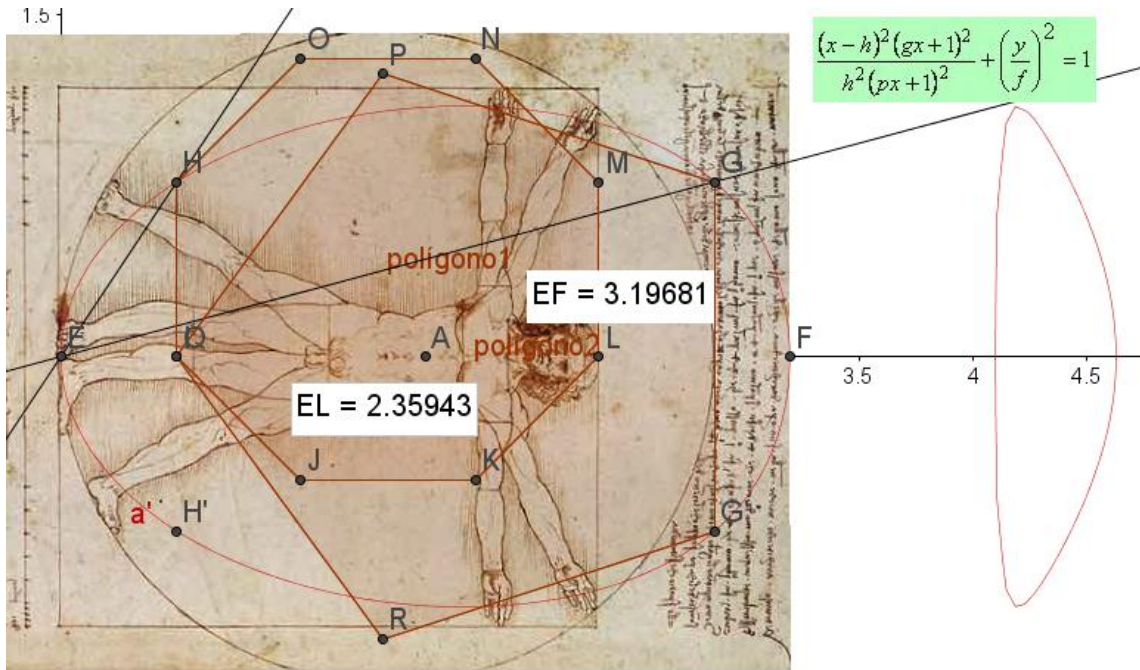
The first egg is by mr Yamamoto. The necessary condition practically does not change the values that Nobuo Yamamoto gives us for an egg shape curve that seems a hen's egg. In other hand the human height is one of the lower values that we'll see, near 0.005 lower than average height.



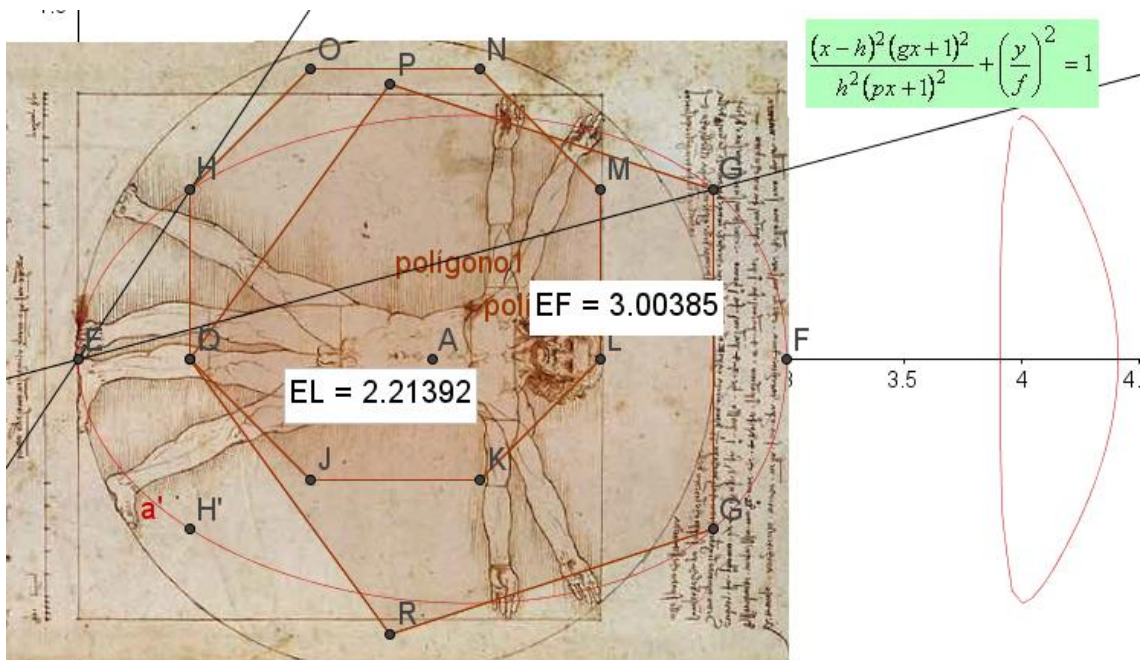
**Itou egg major axis is 1 and human height is 0.73439
Yamamoto gives us a=0.5 b=0.39 practically the same
that with necessary condition (a=0.5 b=0.39095)**

The second egg is by mr Itou. The necessary condition practically does not change the values that Nobuo Yamamoto gives us for an egg shape curve that seems a hen's egg. The human height is higher than in Yamamoto egg and lower (near 0.002) than average human height.

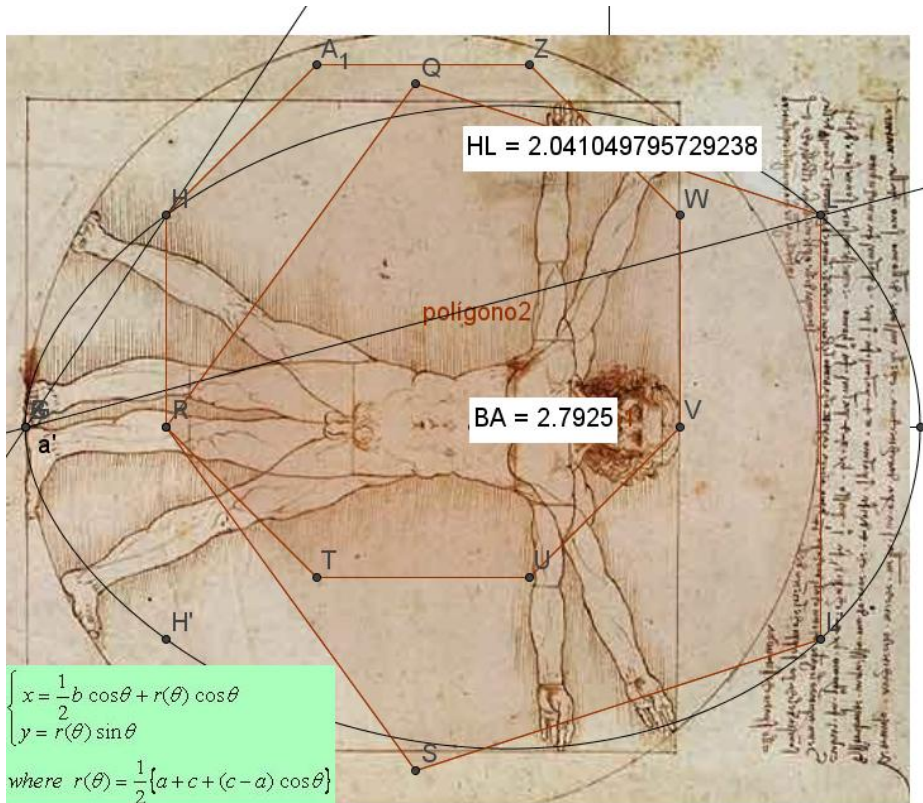
The third egg is by mr Yasuyuki Asai. Of the 4 variables, the necessary condition change the variable f or variable h in this case in greater amount than the first and second curves (always since the values that Yamamoto give us for a curve that seems a hen's egg. The human height is higher (near 0.002 and 0.001) than average human height.



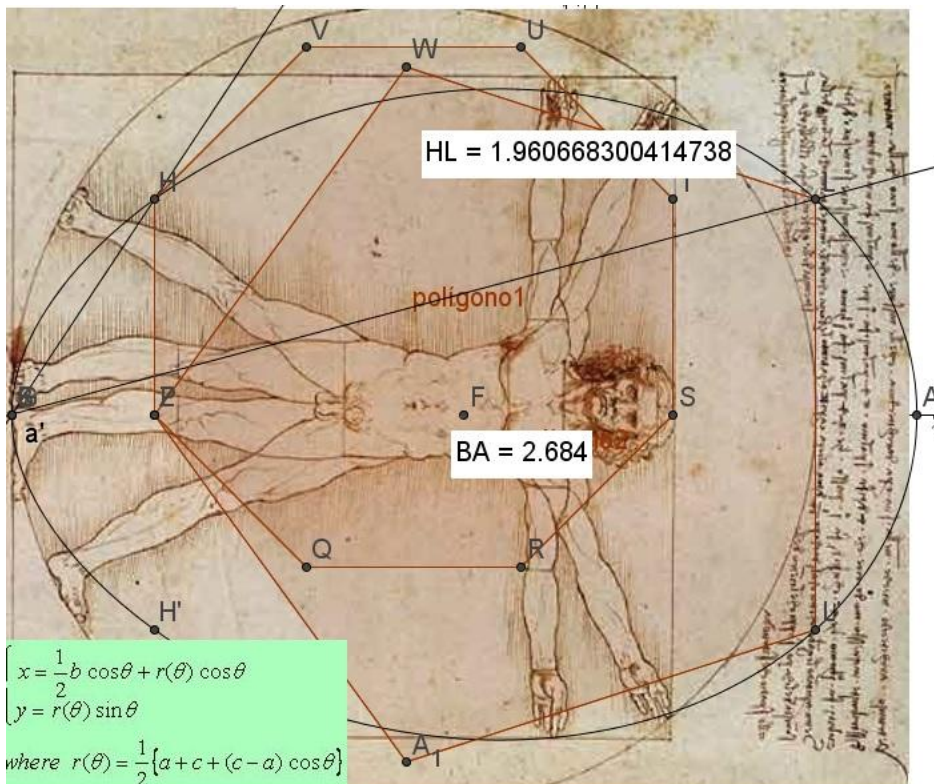
the Yasuyuki Asai egg major axis is 3.19681 and the human height 2.35943 for $f=1.1$ $g=1$ $h=1.4345$ $p=1.3$ Yamamoto give us values $f=1.1$ $g=1$ $h=1.35$ $p=1.3$, if major axis is 1 then human height is $2.35943/3.19681 = 0.73792$



the Yasuyuki Asai egg major axis is 3.00385 and the human height 2.21392 for $f=1.0625$ $g=1$ $h=1.35$ $p=1.3$ Yamamoto give us values $f=1.1$ $g=1$ $h=1.35$ $p=1.3$, if major axis is 1 then human height is $2.21392/3.00385 = 0.73709$

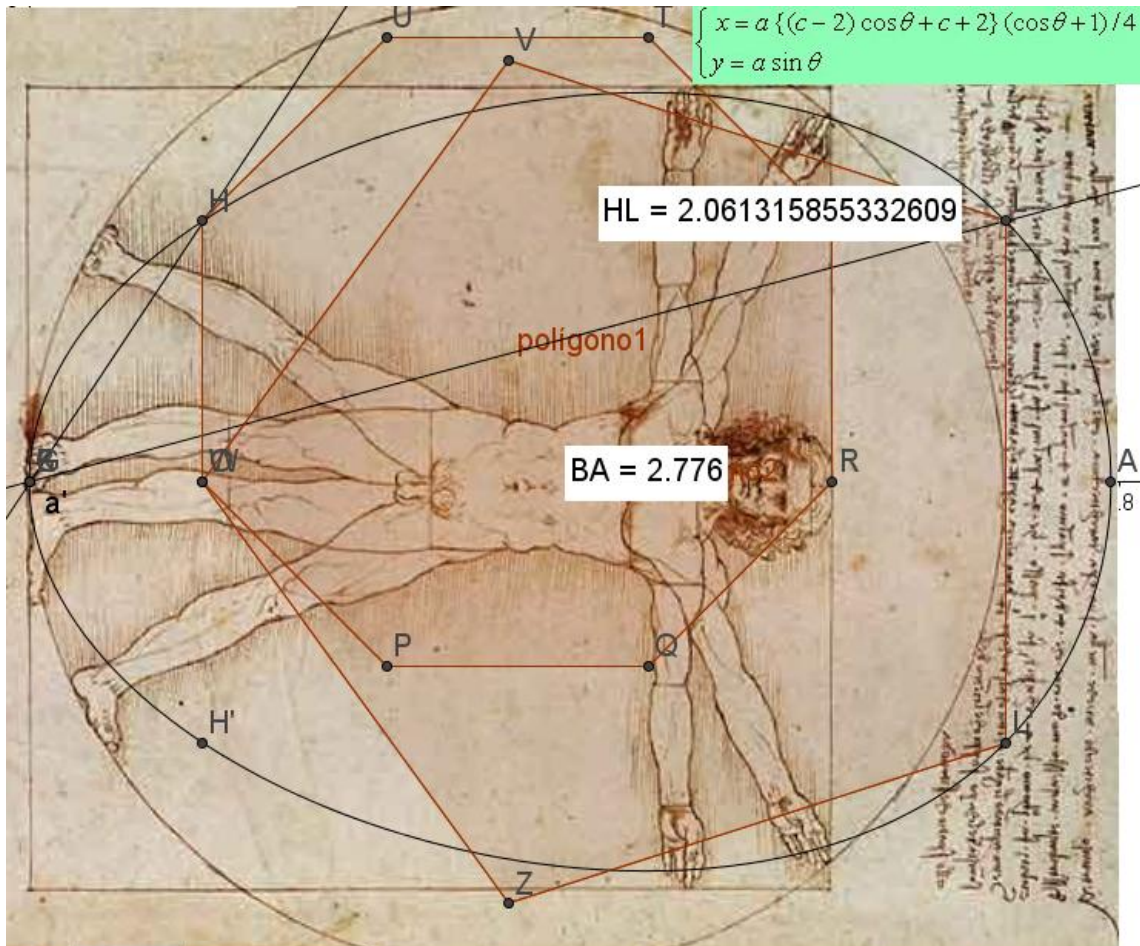


the 2.7925 Asai egg major axis if it was 1 then the human height is $2.041049795729238/2.7925=0.73090413023666$ (lower than first egg) with $a=1.35$ $b=0.9425$ $c=0.5$ while Yamamoto give us $a=1.35$ $b=0.9$ $c=0.5$



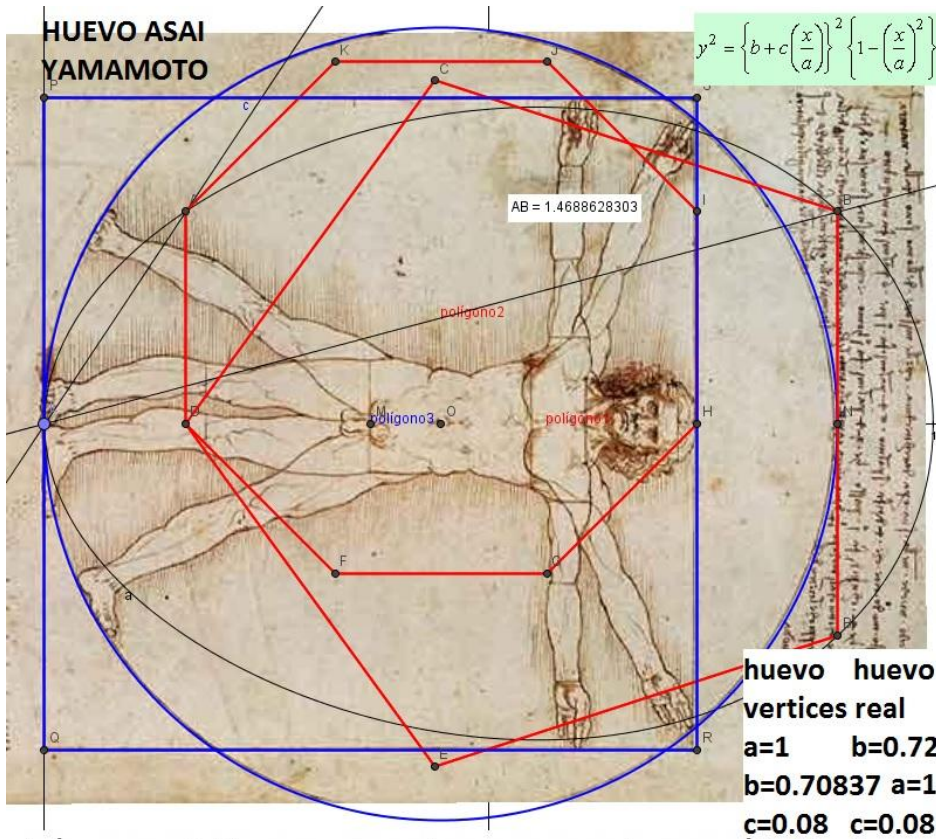
the 2.684 Asai egg major axis if it was 1 then the human height is $1.96066830041473 / 2.684 = 0,73050234757132$ (lower than first egg) with $a=1.284$ $b=0.9$ $c=0.5$ while Yamamoto give us $a=1.35$ $b=0.9$ $c=0.5$

The sixth egg is by mr Asai too. The equation has 2 variables and with the necessary condition I change c variable from 2.9 to 2.776. Here the human height is higher than average human height and higher than the other egg curves.

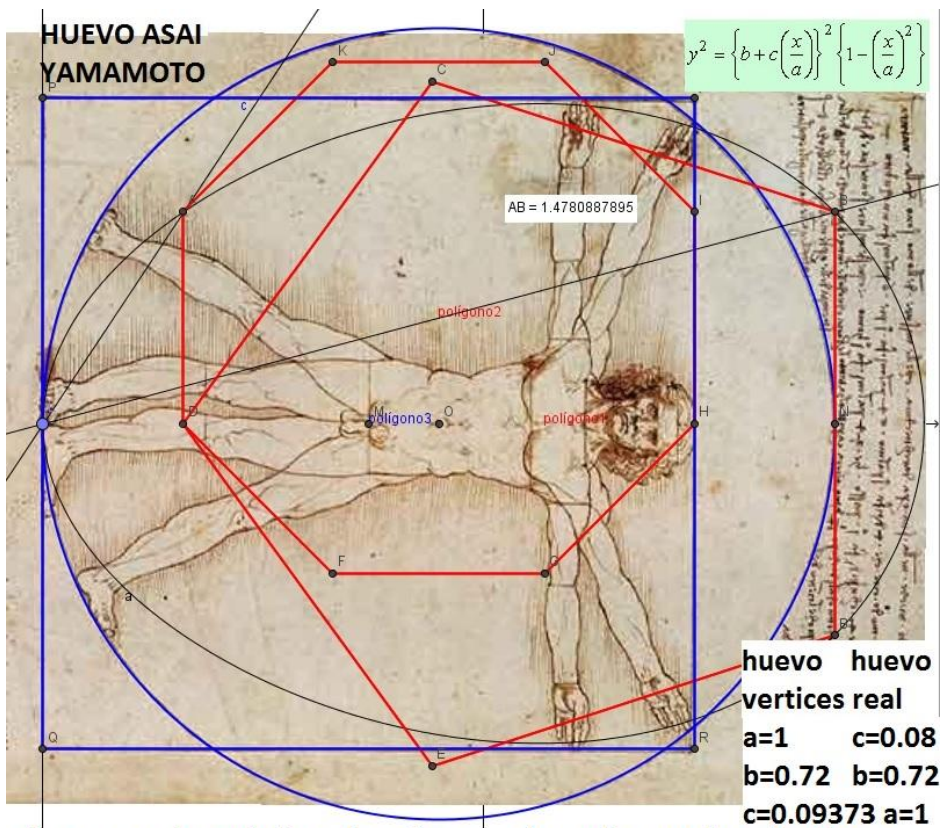


the 2.776 Asai egg major axis if it was 1 then the human height is 2.061315855332609 / 2.776 = 0,74254893924085 (higher than average) with a=1 c=2.776 while Yamamoto give us a=1 c=2.9 for the hen's egg

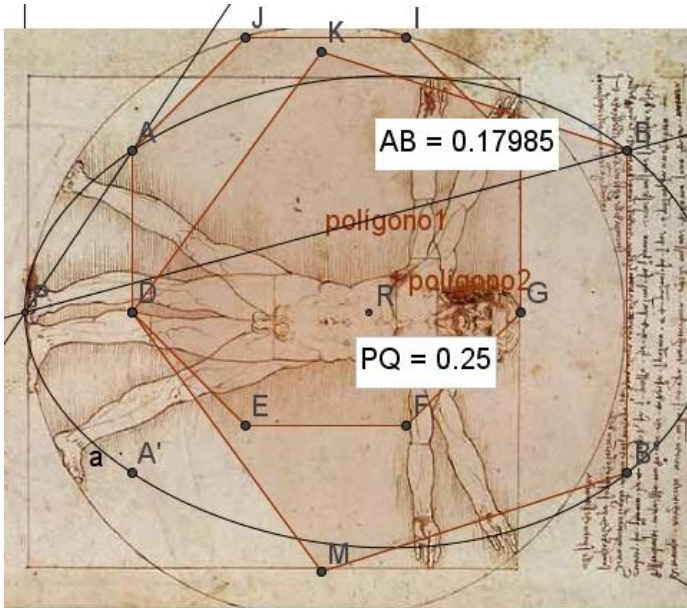
The seventh egg is by mr Asai too, the human height changing b is 0.002 lower than average height and changing c is 0.003 higher than average height, then changing the both variables the height must be close average height



the egg major axis is 2 then for a major axis equals 1 the human height is $1.46886/2=0.734431$ we change b from 0.72 to 0.70837

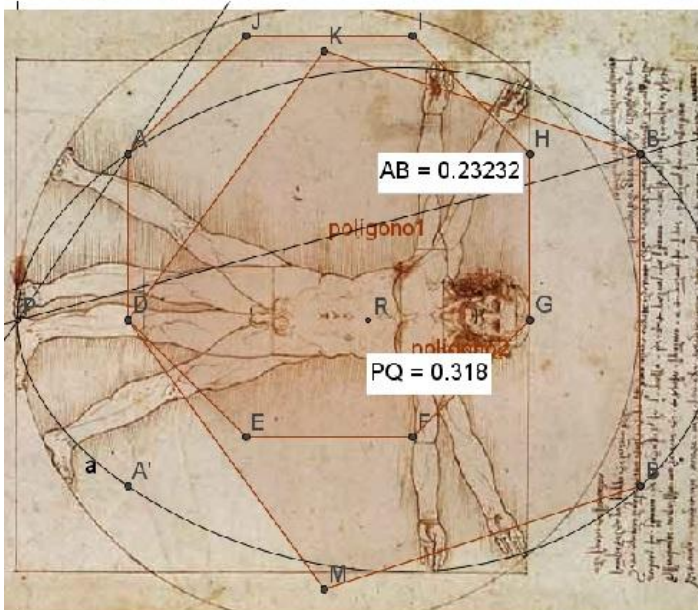
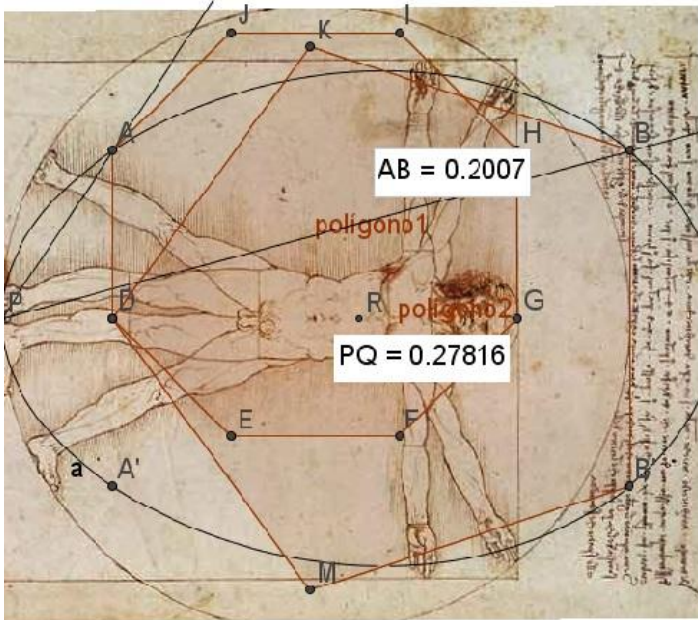


the egg major axis is 2 then for a major axis equals 1 the human height is $1.47808/2=0.739044$ we change c from 0.08 to 0.09373



$$k_2 e^{-b^2 x^2} + e^{-c^2 y^2} = k_1 e^{-a^2} + k_3 e^{-dx}$$

Yamamoto give us $a=0$
 $b=2$ $c=2.65$
 $d=1$ for the hen's egg,
 for the necessary condition we have 3 eggs
 the first with $c=2.829$
 the second with $b=1.896$
 the third with $d=1.272$
 the human heighth for a egg major axis equals 1
 in the first egg is 0.7194
 the second egg is 0.7215
 the third egg 0.7305



all the human heighth are the lowest of all equations we have seen and the first and second are lower than minimum height for hugelschafer

In conclusion: only the second egg of Yamamoto has not the human height inside the extreme values of the huggelschaffer egg but some changings of this egg are included inside the extreme values. The forth and the seven egg (the both by Asai) have the human height closer to the average. The other eggs some are higher and some are lower than the average height. There is some eggs where the changes are very small: the first (by Yamamoto) the second (by Itou) and the forth (by Asai)

I think this equations give us an human height close to an hypothetical egg passing on the vertexes of the octagon and pentagon of the anatomical canon where we have the rational distances $\frac{3}{8}$ $\frac{5}{8}$ square root of $\frac{1}{12}$ and square root of $\frac{1}{5}$ and where the major axis of the egg is 1.

Ignacio Colmenero Vargas